



[1]Solve the following P.D.E:

(14)

(a)  $4u_x + 3u_y + 10u = 10x$

(b)  $u_{xx} - 4u_{xy} - 5u_{yy} = x^2y^2$

[2](a)Show that the function  $v(x, y) = x^3 - 3xy^2 + 2$  is harmonic and find its conjugate  $u(x, y)$  such that the function  $w = f(z) = u + iv$  is analytic.

(4)

(b)Determine and sketch the image of the region  $G: 0 \leq x \leq \pi/2, 0 \leq y \leq 4$  under the function  $f(z) = \sin z$ .

(4)

(c)Find the integral  $\oint_C \frac{\cos z}{(2z-\pi)^2} dz$ ,  $C$  is  $|z| = 3$

(4)

(d)By Residue theorem, obtain the integral  $\int_0^{\infty} \frac{\cos 2x}{x^2+1} dx$

(4)

[3](a)Solve the following linear system, number of iterations is 3:

(8)

$$2x_1 - x_2 - x_3 + x_4 = 2, \quad x_1 + 2x_2 - x_3 - x_4 = 2,$$

$$x_1 - x_2 + 3x_3 - 2x_4 = 2, \quad x_1 - x_2 + 2x_3 - x_4 = 2$$

(b)Using trapezoidal rule, compute the integral  $\int_1^{\infty} \frac{\sqrt{\sin(1/x)}}{x^2} dx$ ,  $\Delta = 0.2$

(4)

[4]From the data (1, 2), (3, 4), (4, 10), (5, 8):

(15)

(a)Find the exponential curve that fits the data.

(b)Write the table of differences of the data and obtain the value of  $y$  at  $x = 0$ .

(c)Write the Lagrange's polynomial of degree 2 to obtain the value of  $x$  at  $y = 3$ .

[5](a)Solve the LP problem: maximize  $f = x_1 - x_2 + 2x_3 + x_4$

(6)

$$\text{s.t } x_1 + x_2 + x_3 - x_4 \leq 10, \quad x_1 + x_2 + x_3 - 2x_4 \leq 8,$$

$$x_1 + x_2 - x_3 + 2x_4 \leq 6, \quad x_1, x_2, x_3, x_4 \geq 0$$

(b)Write and solve the dual problem of the LP problem:

(7)

$$\text{minimize } f = 3x + y$$

$$\text{s.t } x - y \leq 4, \quad -x + y \leq 1, \quad x + y \geq 3, \quad x, y \geq 0$$